Questions and Solutions

PART- A : MATHEMATICS

1. If S is the set of distinct values of 'b' for which the following system of linear equations $x + y + z = 1$, $x + ay + z = 1$, $ax + by + z = 0$ has no solution, then S is :
(1) an empty set
(2) an infinite set
(3) a finite set containing two or more elements
(4) a singleton

1. (4)

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = -(a-1)^2$$

$\Delta_x = (a-1)$
$\Delta_y = 0$
$\Delta_z = -a(a-1)$
If $a = 1 \Rightarrow \Delta = \Delta_x = \Delta_y = \Delta_z = 0$
$\Rightarrow$ Infinite solution
For $b = 1$ & $a = 1 \Rightarrow x + y + z = 1$
$x + y + z = 0$
$\Rightarrow$ no solution
Only one value of b.

2. The following statement $(p \to q) \to [(\sim p \to q) \to q]$ is :
(1) a tautology
(2) equivalent to $\sim p \to q$
(3) equivalent to $p \to \sim q$
(4) a fallacy

2. (1)
$(p \to q) \to [(\sim p \to q) \to q]$ is

1	2	3	4	5	6	7
p	q	~p	$p \to q$	$\sim p \to q$	$[(\sim p \to q) \to q]$	$4 \to 6$
T	T	F	T	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	F	T	T	F	T	T

3. If $5\left(\tan^2 x - \cos^2 x\right) = 2\cos 2x + 9$, then the value of $\cos 4x$ is :
(1) $-\frac{3}{5}$
(2) $\frac{1}{3}$
(3) $\frac{2}{9}$
(4) $-\frac{7}{9}$

3. (4)

$$5\left(\tan^2 x - \cos^2 x\right) = 2\cos 2x + 9$$

$$5\left(\sec^2 x - 1 - \cos^2 x\right) = 2(2\cos^2 x - 1) + 9$$

$$5\left(\frac{1}{\cos^2 x} - 1 - \cos^2 x\right) = 4\cos^2 x - 2 + 9$$

$$5\left(\frac{1-\cos^2 x-\cos^4 x}{\cos^2 x}\right) = 4\cos^2 x + 7$$

$5 - 5\cos^2 x - 5\cos^4 x = 4\cos^4 x + 7\cos^2 x$

$\therefore\ 9\cos^4 x + 12\cos^2 x - 5 = 0$

Put, $\cos^2 x = m$

$\therefore\ 9m^2 + 12m - 5 = 0$

$\therefore\ 9m^2 + 15m - 3m - 5 = 0$

$3m(3m + 5) - 1(3m + 5) = 0$

$\therefore\ (3m + 5)(3m - 1) = 0$

$\therefore\ m = \frac{1}{3}$ or $m = -\frac{5}{3}$

$\therefore\ \cos^2 x = \frac{1}{3}$ or $\cos^2 x = -\frac{5}{3}$

But, $\cos^2 x \neq -\frac{5}{3}$

$\therefore\ \cos^2 x = \frac{1}{3}$

Now, $\cos 4x = 2\cos^2 2x - 1$

$= 2(\cos 2x)^2 - 1$

$= 2(2\cos^2 x - 1)^2 - 1$

$= 2\left[2\times\frac{1}{3}-1\right]^2 - 1 = 2\left[\frac{2}{3}-1\right]^2 - 1 = 2\left[-\frac{1}{3}\right]^2 - 1$

$= \frac{2}{9} - 1 = \frac{2-9}{9} = -\frac{7}{9}$

4. For three events A, B and C, P (Exactly one of A or B occurs) = P (Exactly one of B or C occurs) = P (Exactly one of C or A occurs) $= \frac{1}{4}$ and P (All the three events occur simultaneously $= \frac{1}{16}$.

Then the probability that at least one of the events occurs is :

(1) $\frac{7}{32}$ (2) $\frac{7}{16}$ (3) $\frac{7}{64}$ (4) $\frac{3}{16}$

4. (2)

P(exactly one of A or B occurs) $= \frac{1}{4} \Rightarrow P(A) + P(B) - 2P(A \cap B) = \frac{1}{4}$

P(exactly one of B & C occurs) $= \frac{1}{4} \Rightarrow P(B) + P(C) - 2P(B \cap C) = \frac{1}{4}$

P(exactly one of C or A occurs) $= \frac{1}{4} \Rightarrow P(C) + P(A) - 2P(A \cap C) = \frac{1}{4}$

$P(A \cap B \cap C) = \frac{1}{16}$

Ad. $[P(A) + P(B) + (C) - P(A \cap B) - P(B \cap C) - P(C \cap A)] = \frac{3}{8}$ …. (1)

Probability of at least one event

$= P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$= \frac{3}{8} + \frac{1}{16} = \frac{6+1}{16} = \frac{7}{16}$

5. Let ω be a complex number such that 2ω + 1 = z, where z = $\sqrt{-3}$. If $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$,

then k is equal to :

(1) −z (2) z (3) −1 (4) 1

5. (1)

$w = \frac{\sqrt{3}i-1}{2}$ $w = \frac{-1+\sqrt{3}i}{2}, w^2 = \frac{-1-\sqrt{3}i}{2}$

$1 + w + w^2 = 0, w^3 = 1$

$\therefore \quad 3k = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -w^2-1 & w^2 \\ 1 & w^2 & w^7 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & w & w^2 \\ 1 & w^2 & w \end{vmatrix}$

$= 1(w^2 - w^4) - 1(w - w^2) + 1(w^2 - w)$

$= w^2 - w^4 - w + w^2 + w^2 - w$

$= 3w^2 - 2w - w^4$

$= 3w^2 - 2w - w = 3(w^2 - w)$

$\therefore \quad k = w^2 - w = \frac{-1-\sqrt{3}i}{2} - \frac{-1+\sqrt{3}i}{2}$

$= \frac{-1-\sqrt{3}i+1-\sqrt{3}i}{2} = \frac{-2\sqrt{3}i}{2} = -\sqrt{3}i = -z$

$(\because 2w + 1 = z = \sqrt{3}i)$

6. Let k be an integer such that the triangle with vertices (k, −3k), (5, k) and (−k, 2) has area 28 sq. units. Then the orthocenter of this triangle is at the point :

(1) $\left(2, -\frac{1}{2}\right)$ (2) $\left(1, \frac{3}{4}\right)$ (3) $\left(1, -\frac{3}{4}\right)$ (4) $\left(2, \frac{1}{2}\right)$

6. (4)

$\Delta = \frac{1}{2}\left|(k^2 + 10 + 3k^2) - (-15k - k^2 + 2k)\right| = 28$

$\left|5k^2 + 13k + 10\right| = 56$

$5k^2 + 13k + 10 = \pm 56$

$5k^2 + 13k + 10 = 56$

$5k^2 + 13k - 46 = 0$

$5k^2 + 23k - 10k - 46 = 0$

$k(5k + 23) - 2(5k + 23) = 0$

$k = 2, -\frac{23}{5}$

k = 2 as integer

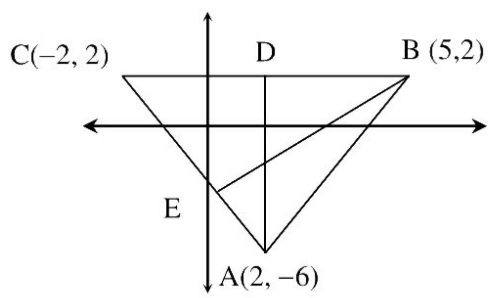
A ≡(2, −6), B ≡ (5, 2) , C ≡ (−2, 2).

As BC perpendicular to x−axis

⇒ eqn. of altitude AD is x = 2

For BE, $y - 2 = \frac{1}{2}(x - 5)$

Solving orthocentre $\left(2, \frac{1}{2}\right)$



7. Twenty meters of wire is available for fencing off a flower–bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower–bed is :

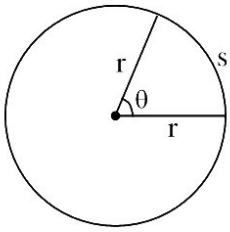
(1) 12.5 (2) 10 (3) 25 (4) 30

7. (3)

$2r + s = 20$ (length of wire)

Now $s = r\theta$

$\therefore\ 2r + r\theta = 20 \qquad \therefore\ \theta = \dfrac{20-2r}{r}$



$\therefore\ A = \dfrac{1}{2}r^2\theta = \dfrac{1}{2}r^2\left(\dfrac{20-2r}{r}\right)$

$A = 10\,r - r^2$

$\therefore\ \dfrac{dA}{dr} = 10 - 2r = 0 \Rightarrow r = 5$

$\dfrac{d^2A}{dr^2} = -2 < 0 \Rightarrow$ Area is maximum

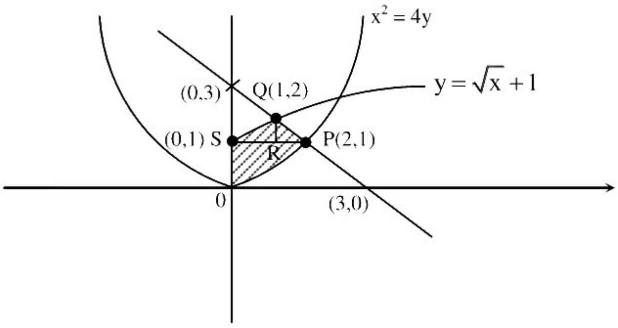
$\therefore$ Maximum Area $= \dfrac{1}{2}r^2\theta = \dfrac{1}{2}\times 25\times\dfrac{20-2r}{r}$

$= \dfrac{1}{2}\times 25\times\dfrac{20-10}{5} = \dfrac{1}{2}\times 5\times 10 = 25$ sq.m.

8. The area (in sq. units) of the region $\left\{(x, y): x \ge 0, x + y \le 3, x^2 \le 4y \text{ and } y \le 1 + \sqrt{x}\right\}$ is :

(1) $\dfrac{59}{12}$ (2) $\dfrac{3}{2}$ (3) $\dfrac{7}{3}$ (4) $\dfrac{5}{2}$

8. (4)



Area $= \int_0^1 x\,dy + \int_0^1 y\,dx + \Delta PQR$

$= \int_0^1 2\sqrt{y}\,dy + \int_0^1 \sqrt{x}\,dx + \dfrac{1}{2}1\times 1$

$= 2\times\dfrac{2}{3} + \dfrac{2}{3} + \dfrac{1}{2} = 2 + \dfrac{1}{2} = \dfrac{5}{2}$

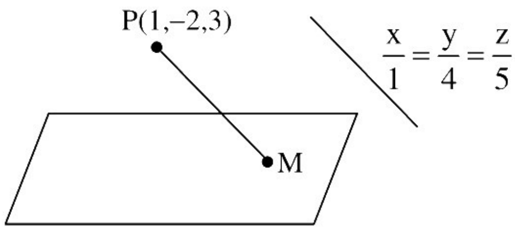
9. If the image of the point P (1, −2, 3) in the plane, $2x + 3y - 4z + 22 = 0$, measured parallel to the line, $\dfrac{x}{1} = \dfrac{y}{4} = \dfrac{z}{5}$ is Q, then PQ is equal to :

(1) $3\sqrt{5}$ (2) $2\sqrt{42}$ (3) $\sqrt{42}$ (4) $6\sqrt{5}$

9. (2)

Line PM (Parallel to given line) is

$\dfrac{x-1}{1} = \dfrac{y+2}{4} = \dfrac{z-3}{5} = r(\text{Let})$



$M \equiv (r + 1, 4r - 2, 5r + 3)$

M satisfy plane, $2x + 3y - 4z + 22 = 0$

$\Rightarrow 2r + 2 + 12r - 6 - 20r - 12 + 22 = 0$

$-6r + 6 = 0$

$r = 1$

So, M = (2, 2, 8)

$PM = \sqrt{(2-1)^2 + (2+2)^2 + (8-3)^2} = \sqrt{1+16+25} = \sqrt{42}$

So, $PQ = 2\sqrt{42}$

10. If for $x \in \left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}\,.\,g(x)$, then g(x) equals :

(1) $\frac{9}{1+9x^3}$ (2) $\frac{3x\sqrt{x}}{1-9x^3}$ (3) $\frac{3x}{1-9x^3}$ (4) $\frac{3}{1+9x^3}$

10. (1)

$$y = \tan^{-1}\left[\frac{2\cdot\left(3x\sqrt{x}\right)}{1-\left(3x\sqrt{x}\right)^2}\right]. \quad \text{Let } 3x\sqrt{x} = \tan\theta$$

$$= \tan^{-1}(\tan 2\theta) = 2\theta = 2\tan^{-1}\left(3x\sqrt{x}\right)$$

$$\frac{dy}{dx} = 2\cdot\frac{1}{1+9x^3}\; 3\cdot\frac{3}{2}\; x^{\frac{1}{2}} = \frac{9\sqrt{x}}{1+9x^3} = \sqrt{x}\cdot\frac{9}{1+9x^3}$$

$$\Rightarrow g(x) = \frac{9}{1+9x^3}$$

11. If $(2+\sin x)\frac{dy}{dx} + (y+1)\cos x = 0$ and y(0) = 1, then $y\left(\frac{\pi}{2}\right)$ is equal to :

(1) $\frac{1}{3}$ (2) $-\frac{2}{3}$ (3) $-\frac{1}{3}$ (4) $\frac{4}{3}$

11. (1)

$$\frac{dy}{dx} + \frac{y\cos x}{2+\sin x} + \frac{\cos x}{2+\sin x} = 0$$

$$\frac{dy}{dx} + \frac{\cos x}{2+\sin x}y = -\frac{\cos x}{2+\sin x}$$

$$\text{I.F.} = e^{\int\frac{\cos x}{2+\sin x}dx} = 2+\sin x$$

$$y\,.\,(2+\sin x) = c + \int(-\cos x)dx$$

$y(2 + \sin x) = c - \sin x$

Given y (0) = 1

$1(2 + 0) = c - 0$

$c = 2$

Soln. $y(2 - \sin x) = 2 - \sin x$

$$y(\pi/2) = \frac{2-1}{2+1} = \frac{1}{3}$$

12. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If $\angle BPC = \beta$, then $\tan\beta$ is equal to :

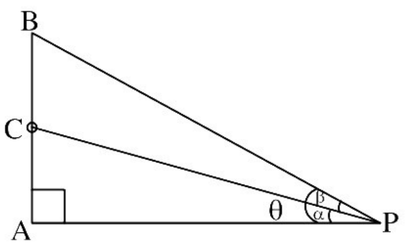
(1) $\frac{6}{7}$ (2) $\frac{1}{4}$ (3) $\frac{2}{9}$ (4) $\frac{4}{9}$

12. (3)

AP = 2AB

$$\tan\theta = \frac{AB}{AP} = \frac{1}{2}$$

$$\Rightarrow \tan\alpha = \frac{AC}{AP} = \frac{\frac{AB}{2}}{2AB} = \frac{1}{4}$$



$\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$

$\dfrac{1}{2} = \dfrac{\frac{1}{4}+\tan\beta}{1-\frac{1}{4}.\tan\beta}$

$\dfrac{1}{2} = \dfrac{1+4\tan\beta}{4-\tan\beta} \quad \Rightarrow\ 2 + 8\tan\beta = 4 - \tan\beta$

$9\tan\beta = 2$

$\tan\beta = \dfrac{2}{9}$

13. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to :

(1) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$ (2) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$ (3) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$ (4) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

13. (2)

$A^2 = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix}$

$3A^2 + 12A = \begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix} + \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix} = \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

Adj. $(3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

14. For any three positive real numbers a, b and c, $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$. Then :

(1) b, c and a are in G.P. (2) b, c and a are in A.P.
(3) a, b and c are in A.P. (4) a, b and c are in G.P.

14. (2)

$225a^2 + 9b^2 + 25c^2 - 75ac - 45ab - 15bc = 0$

$\Rightarrow 450a^2 + 18b^2 + 50c^2 - 150ac - 90ab - 30bc = 0$

$\Rightarrow (15a - 5c)^2 + (15a - 3b)^2 + (3b - 5c)^2 = 0$

$\Rightarrow 15a = 5c \qquad 15a = 3b \qquad 3b = 5c$

$\therefore\ 15a = 3b = 5c$

$\Rightarrow \dfrac{a}{1} = \dfrac{b}{5} = \dfrac{c}{3}$

$\Rightarrow$ b, c, a in A.P.

15. The distance of the point (1, 3, –7) from the plane passing through the point (1, –1, –1) having normal perpendicular to both the lines $\dfrac{x-1}{1} = \dfrac{y+2}{-2} = \dfrac{z-4}{3}$ and $\dfrac{x-2}{2} = \dfrac{y+1}{-1} = \dfrac{z+7}{-1}$ is :

(1) $\dfrac{20}{\sqrt{74}}$ (2) $\dfrac{10}{\sqrt{83}}$ (3) $\dfrac{5}{\sqrt{83}}$ (4) $\dfrac{10}{\sqrt{74}}$

15. (2)

Plane passing (1, –1,–1) is

$a(x - 1) + b(y + 1) + c(z + 1) = 0$

Normal of line containing both the line is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix} = 5\hat{i} + 7\hat{j} + 3\hat{k}$$

From given condition normal of plane parallel to $5\hat{i} + 7\hat{j} + 3\hat{k}$

$\Rightarrow$ Plane is $5(x - 1) + 7(y + 1) + 3(z + 1) = 0$

$5x + 7y + 3z + 5 = 0$

Distance $= \left| \dfrac{5 \times 1 + 7 \times 3 + 3 \times (-7) + 5}{\sqrt{5^2 + 7^2 + 3^2}} \right| = \dfrac{10}{\sqrt{83}}$

16. Let $I_n = \int \tan^n x\, dx, (n > 1)$. If $I_4 + I_6 = a \tan^5 x + bx^5 + C$, where C is a constant of integration, then the ordered pair (a, b) is equal to :

(1) $\left(-\dfrac{1}{5}, 1\right)$ (2) $\left(\dfrac{1}{5}, 0\right)$ (3) $\left(\dfrac{1}{5}, -1\right)$ (4) $\left(-\dfrac{1}{5}, 0\right)$

16. (2)

$I_n = \int \tan^{n-2} x . (\sec^2 x - 1)\, dx$

$= \int \tan^{n-2} x . \sec^2 x\, dx - \int \tan^{n-2} x\, dx$

$I_n = \dfrac{\tan^{n-1} x}{n-1} - I_{n-2}$

$I_n + I_{n-2} = \dfrac{\tan^{n-1} x}{n-1} + C$

$n = 6$

$I_6 + I_4 = \dfrac{\tan^5 x}{5} + C$

Equate $a = \dfrac{1}{5}, b = 0$

$(a, b) = \left(\dfrac{1}{5}, 0\right)$

17. The eccentricity of an ellipse whose centre is at the origin is $\dfrac{1}{2}$. If one of its directrices is $x = -4$, then the equation of the normal to it at $\left(1, \dfrac{3}{2}\right)$ is :

(1) $2y - x = 2$ (2) $4x - 2y = 1$ (3) $4x + 2y = 7$ (4) $x + 2y = 4$

17. (2)

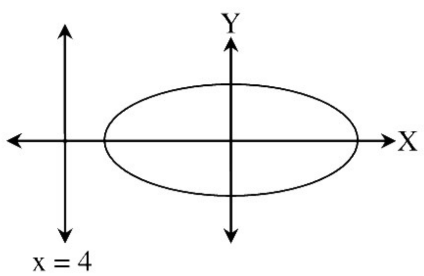
$e = \dfrac{1}{2}$

$x = -\dfrac{a}{e}$

$-4 = -\dfrac{a}{\frac{1}{2}}$

$4 = 2a$

$a = 2$



$e^2 = 1 - \frac{b^2}{a^2}$

$\frac{1}{4} = 1 - \frac{b^2}{4}$

$\frac{b^2}{4} = 1 - \frac{1}{4}$

$b^2 = 3$

ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$

$\frac{2x}{4} + \frac{2y}{3}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{2x}{4} \times \frac{3}{2y} = -\frac{3x}{4y}$

$m_{tangent} = -\frac{3 \times 1}{4 \times 3} = -\frac{1}{2}$

Normal at $\left(1, \frac{3}{2}\right)$ is

$y - \frac{3}{2} = 2(x - 1)$

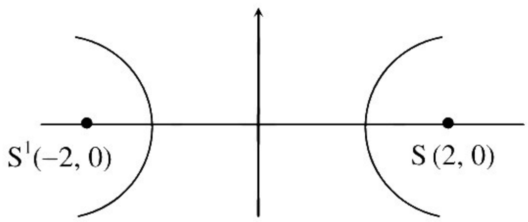
$2y - 3 = 4x - 4$

$4x - 2y - 1 = 0$

18. A hyperbola passes through the point $P\left(\sqrt{2}, \sqrt{3}\right)$ and has foci at (± 2, 0). Then the tangent to this hyperbola at P also passes through the point :

(1) $\left(3\sqrt{2}, 2\sqrt{3}\right)$ (2) $\left(2\sqrt{2}, 3\sqrt{3}\right)$ (3) $\left(\sqrt{3}, \sqrt{2}\right)$ (4) $\left(-\sqrt{2}, -\sqrt{3}\right)$

18. (2)



foci $S \equiv (\pm ae, 0)$

$2ae = 4$

$a^2e^2 = 4$

$a^2\left(1 + \frac{b^2}{a^2}\right) = 4$

$a^2 + b^2 = 4$

$a^2 = 4 - b^2$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

It passes P $\left(\sqrt{2}, \sqrt{3}\right)$

$\frac{2}{a^2} = \frac{3}{b^2} = 1$

$\frac{2}{4 - b^2} - \frac{3}{b^2} = 1$

$2b^2 - 12 + 3b^2 = b^2(4 - b^2)$

$b^4 + b^2 - 12 = 0$

$(b^2 + 4)(b^2 - 3) = 0$

$b^2 = 3$

$\Rightarrow a^2 = 1$

Hyperbola $\frac{x^2}{1} - \frac{y^2}{3} = 1$

Tangent at $\left(\sqrt{2}, \sqrt{3}\right)$ is $x\sqrt{2} - \frac{y\sqrt{3}}{3} = 1$

Obviously $\left(2\sqrt{2}, 3\sqrt{3}\right)$ satisfy

19. The function $f : R \to \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$, is :

(1) invertible
(2) injective but not surjective
(3) surjective but not injective
(4) neither injective nor surjective

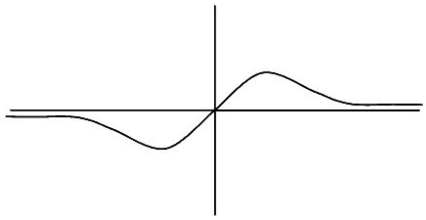
19. (3)

Obviously from graph not one–one.

$$f'(x) = \frac{1-x^2}{(1+x^2)^2} < 0 \qquad \forall\ x \in R$$

$\Rightarrow$ f(x) $\downarrow$

Obviously range $\left[-\frac{1}{2}, \frac{1}{2}\right]$.



20. $\lim\limits_{x \to \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals :

(1) $\frac{1}{24}$
(2) $\frac{1}{16}$
(3) $\frac{1}{8}$
(4) $\frac{1}{4}$

20. (2)

Let $L = \lim\limits_{x \to \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$

Put, $\pi - 2x = \theta \quad \Rightarrow \quad \pi - \theta = 2x \quad \Rightarrow \quad x = \frac{\pi}{2} - \frac{\theta}{2}$

As $x \to \frac{\pi}{2}, \theta \to 0$

$$\therefore \quad L = \lim_{\theta \to 0} \frac{\cot\left(\frac{\pi}{2} - \frac{\theta}{2}\right) - \cos\left(\frac{\pi}{2} - \frac{\theta}{2}\right)}{\theta^3}$$

$$= \lim_{\theta \to 0} \frac{\tan\frac{\theta}{2} - \sin\frac{\theta}{2}}{\theta^3} = \lim_{\theta \to 0} \frac{\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} - \sin\frac{\theta}{2}}{\theta^3} = \lim_{\theta \to 0} \frac{\sin\frac{\theta}{2} - \sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2}}{\theta^3 \cos\frac{\theta}{2}}$$

$$= \lim_{\theta \to 0} \frac{\sin\frac{\theta}{2}\left(1 - \cos\frac{\theta}{2}\right)}{\cos\frac{\theta}{2}\,\theta^3} = \lim_{\theta \to 0} \frac{\sin\frac{\theta}{2}}{\frac{\theta}{2}} \times \frac{1}{2} \times \lim_{\theta \to 0} \frac{2\sin^2\frac{\theta}{4}}{\frac{\theta^2}{16}} \times \frac{1}{16} \times \lim_{\theta \to 0} \frac{1}{\cos\frac{\theta}{2}}$$

$$= 1 \times \frac{1}{2} \times 2 \times \frac{1}{16} \times 1 = \frac{1}{16}$$

21. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c} - \vec{a}| = 3$, $|(\vec{a} \times \vec{b}) \times \vec{c}| = 3$ and the angle between $\vec{c}$ and $\vec{a} \times \vec{b}$ be 30°. Then $\vec{a} . \vec{c}$ is equal to :

(1) $\frac{25}{8}$ (2) 2 (3) 5 (4) $\frac{1}{8}$

21. (2)

$\vec{a} \times \vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

$|\vec{a} \times \vec{b}| = 3$

Given : $|(\vec{a} \times \vec{b}) \times \vec{c}| = 3$

$|\vec{a} \times \vec{b}|\,|\vec{c}| \sin 30° = 3$

$3 \times |\vec{c}| \cdot \frac{1}{2} = 3$

$|\vec{c}| = 2$

$|\vec{c} - \vec{a}| = 3$

$(\vec{c} - \vec{a}) \cdot (\vec{c} - \vec{a}) = 9$

$|\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a} \cdot \vec{c} = 9$

$4 + 9 - 2\vec{a} \cdot \vec{c} = 9$

$\vec{a} \cdot \vec{c} = 2$

22. The normal to the curve y (x − 2) (x − 3) = x + 6 at the point where the curve intersects the y–axis passes through the point :

(1) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$ (2) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (3) $\left(\frac{1}{2}, -\frac{1}{3}\right)$ (4) $\left(\frac{1}{2}, -\frac{1}{3}\right)$

22. (2)

y–axis at (0, 1)

$y(x^2 - 5x + 6) = x + 6$

$$y = \frac{x+6}{x^2 - 5x + 6}$$

$$\frac{dy}{dx} = \frac{(x^2 - 5x + 6) \cdot 1 - (x+6)(2x-5)}{(x^2 - 5x + 6)^2}$$

$$\left.\frac{dy}{dx}\right|_{(0,1)} = \frac{6 - 6(-5)}{6^2} = 1$$

Normal at (0, 1) is $y - 1 = -1(x - 0)$

$y + x = 1$

It passes $\left(\frac{1}{2}, \frac{1}{2}\right)$

23. If two different numbers are taken from the set {0, 1, 2, 3, …, 10}; then the probability that their sum as well as absolute difference are both multiple of 4, is :

(1) $\frac{6}{55}$ (2) $\frac{12}{55}$ (3) $\frac{14}{45}$ (4) $\frac{7}{55}$

23. (1)

{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

$$n(s) = {}^{11}C_2 = \frac{11 \times 10}{1 \times 2} = 55$$

Let A be the event that the sum as well as absolute difference of two different numbers taken from given get are both multiple of 4

$2 + 6 = 8$ divisible by 4

$|6 - 2| = 4$ divisible by 4

$2 + 10 = 12$ divisible by 4

$|10-2| = 8$ divisible by 4

$4 + 8 = 12$ divisible by 4

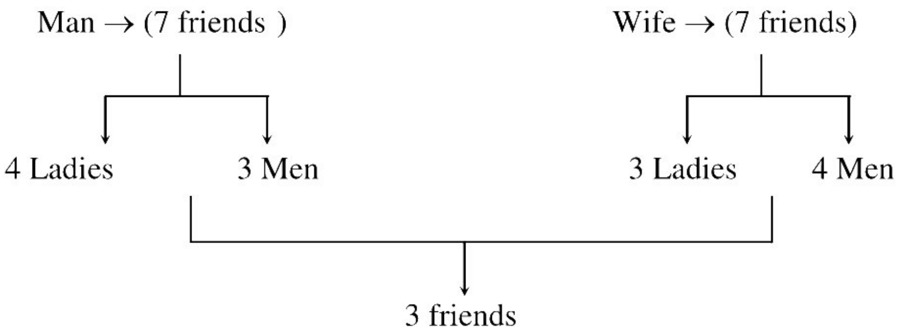
$|8 - 4| = 4$ divisible by 4

$P(A) = \frac{n(A)}{n(S)} = \frac{6}{55}.$

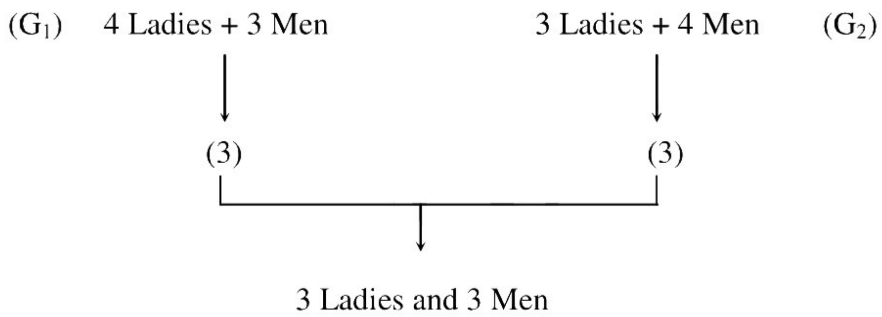
24. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is :

(1) 485 (2) 468 (3) 469 (4) 484

24. (1)



Party Invites → 3 Ladies and 3 Men



G_1	G_2	
3 L	3 M	${}^4C_3 \times {}^4C_3$
3 M	3 L	${}^3C_3 \times {}^3C_3$
1 L and 2 M	2 L and 1 M	${}^4C_1 \times {}^3C_2 \times {}^3C_2 \times {}^4C_1$
2 L and 1 M	1 L and 2 M	${}^4C_2 \times {}^3C_1 \times {}^3C_1 \times {}^4C_2$

Req. No. $= \left({}^4C_3 \times {}^4C_3\right) + \left({}^3C_3 \times {}^3C_3\right) + \left({}^4C_1 \times {}^3C_2 \times {}^3C_2 \times {}^4C_1\right) + \left({}^4C_2 \times {}^3C_1 \times {}^3C_1 \times {}^4C_2\right)$

$= (4\times 4) + (1\times 1) + (4\times 3\times 3\times 4) + \left(\frac{4\times 3}{2}\times 3\times 3\times \frac{4\times 3}{2}\right)$

$= 16 + 1 + 144 + 324 = 485$

25. The value of

$\left({}^{21}C_1 - {}^{10}C_1\right) + \left({}^{21}C_2 - {}^{10}C_2\right) + \left({}^{21}C_3 - {}^{10}C_3\right) + \left({}^{21}C_4 - {}^{10}C_4\right) + \ldots + \left({}^{21}C_{10} - {}^{10}C_{10}\right)$ is :

(1) $2^{21} - 2^{11}$ (2) $2^{21} - 2^{10}$ (3) $2^{20} - 2^{9}$ (4) $2^{20} - 2^{10}$

25. (4)

$\left({}^{21}C_1 + {}^{21}C_2 + \ldots\ldots {}^{21}C_{10}\right) - \left({}^{10}C_1 + {}^{10}C_2 + \ldots\ldots + {}^{10}C_{10}\right)$

$= (2^{20} - 1) - (2^{10} - 1)$

$= 2^{20} - 2^{10}$

As ${}^{21}C_0 + \left({}^{21}C_1 + {}^{21}C_2 + \ldots. + {}^{21}C_{10}\right) + \left({}^{21}C_{11} + \ldots\ldots + {}^{21}C_{20}\right) + {}^{21}C_{21} = 2^{21}$

$2\left({}^{21}C_1 + {}^{21}C_2 + \ldots\ldots + {}^{21}C_{10}\right) = 2^{21} - 2$

${}^{21}C_1 + {}^{21}C_2 + \ldots\ldots + {}^{21}C_{10} = 2^{20} - 1$

26. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one–by–one, with replacement, then the variance of the number of green balls drawn is :

(1) $\frac{12}{5}$ (2) 6 (3) 4 (4) $\frac{6}{25}$

26. (1)

G - 15, Y - 10

P(Green ball in each step) = $\frac{15}{25} = \frac{3}{5}$

P(Yellow ball in each step) = $\frac{2}{5}$

Variance = npq

$= 10 \times \frac{3}{5} \times \frac{2}{5} = \frac{12}{5}$

27. Let a, b, c $\in$ R. If $f(x) = ax^2 + bx + c$ is such that $a + b + c = 3$ and $f(x + y) = f(x) + f(y) + xy$, $\forall$ x, y $\in$ R, then $\sum_{n=1}^{10} f(n)$ is equal to :

(1) 330 (2) 165 (3) 190 (4) 255

27. (1)

$f(x) = ax^2 + bx + c$

$f(1) = a + b + c \quad = 3$

$f(1 + 1) \quad = 2\,f(1) + 1$

$f(2) = 2 \times 3 + 1 \quad = 7$

$f(2 + 1) \quad = f(2) + f(1) + 2$

$f(3) = 7 + 3 + 2 \quad = 12$

$f(3 + 1) \quad = f(3) + f(1) + 3$

$= 12 + 3 + 3 \quad = 18$

$S_n = 3 + 7 + 12 + 18 + \ldots\ldots\ldots + t_n$

$\_\, S_n = \_\, 3 \pm 7 \pm 12 + \ldots\ldots\ldots \pm t_{n-1} \pm t_n$

$0 = (3 + 4 + 5 + 6 + \ldots\ldots\ldots\ldots + n \text{ terms}) - t_n$

$t_n = \frac{n}{2}[6 + (n-1)] = \frac{n}{2}[n+5] = \frac{n^2 + 5n}{2}$

$\sum_{n=1}^{10} f(n) = \frac{1}{2}\left[\sum_{n=1}^{10} n^2 + 5\sum_{n=1}^{10} n\right] = \frac{1}{2}\left[\frac{10\,(11)\,(21)}{6} + 5 \times \frac{10\,(11)}{2}\right] = \frac{1}{2}[55 \times 7 + 55 \times 5]$

$= \frac{55 \times 12}{2} = 330$

28. The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines, $y = |x|$ is :

(1) $2(\sqrt{2}+1)$ (2) $2(\sqrt{2}-1)$ (3) $4(\sqrt{2}-1)$ (4) $4(\sqrt{2}+1)$

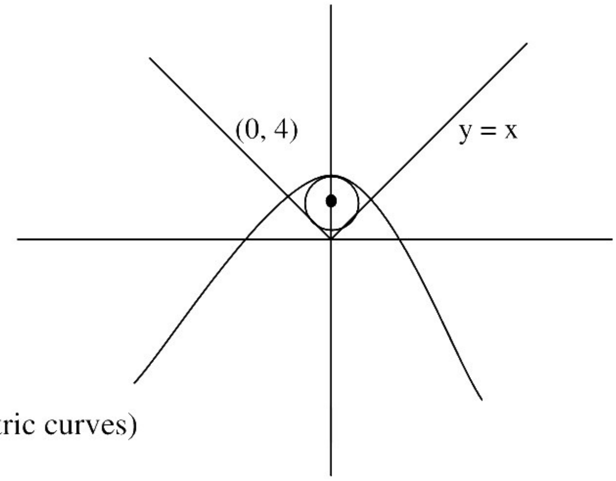
28. (3)

Let centre of circle (0, k)

As touches $x - y = 0$ line

$\Rightarrow \left|\frac{0-k}{\sqrt{2}}\right| = r$ … (radius)

$r^2 = \frac{k^2}{2}$



Circle is $(x - 0)^2 + (y - k)^2 = \frac{k^2}{2}$

As touches $y = 4 - x^2$

$\Rightarrow$ It passes (0, 4) … (as symmetric curves)

$\Rightarrow 0 + (4 - k)^2 = \frac{k^2}{2}$

$\Rightarrow k^2 - 16k + 32 = 0$

$k = 8 \pm 4\sqrt{2}$

Radius for minimum radius

$= 4-(8-4\sqrt{2}) = 4\sqrt{2} - 4 = 4(\sqrt{2}-1)$

29. If for a positive integer n, the quadratic equation,

$x(x+1)+(x+1)(x+2)+...+(x+\overline{n-1})(x+n)=10n$ has two consecutive integral solutions, then n is equal to :

(1) 12 (2) 9 (3) 10 (4) 11

29. (4)

Let the roots be α, $\alpha + 1$

$\alpha(\alpha + 1) + (\alpha + 1)(\alpha + 2) \dots + (\alpha + (n - 1)(\alpha + n) = 10n$

and $(\alpha + 1)(\alpha + 2) + (\alpha + 2)(\alpha + 3) + \dots + (\alpha + n)(\alpha + n + 1) = 10n$

Subtract, $\alpha(\alpha + 1) - (\alpha + n)(\alpha + n + 1) = 0$

$\alpha^2 + \alpha - \alpha^2 - (2n + 1)\alpha - n(n + 1) = 0$

$-2n\alpha - n(n + 1) = 0$

$2\alpha = -(n + 1)$

$\alpha = -\frac{n+1}{2}$

So ‘n’ must be odd

$n = 9, \ \alpha = -4$

$n = 11, \alpha = -5$

Put, $\alpha = -4$, $(-4)(-3) + (-3)(-2) + (-2)(-1) + (-1)\,0 + 0 \times 1 + 1 \times 2 + \dots + 4 \times 5$

$= 12 + 6 + 2 + 2 + 6 + 12 + 20$

$= 60 \neq 10n$